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Transverse Coupled Bunch Stability with Landau Damping

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TRANSVERSE COUPLED BUNCH STABILITY WITH LANDAU DAMPING

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Abstract

This note outlines briefly transverse coupled bunch stability of the proposed Main Injector. It discusses stabilizing effects of the Landau damping induced by both the direct Laslett tune spread and the presence of octupoles. Presented analysis follows from the stability diagram formalism.

1. High Intensity Bunch Coalescing - Main Injector

One shall examine the transverse stability limits of a train of M consecutive, high intensity (8×10^{10} ppb), short proton bunches (ε = 0.5 eV-sec), injected from the Booster to the Main Injector (at the energy of 8.9 GeV), accelerated to the energy of 150 GeV, and then coalesced into a long bunch of the longitudinal emittance ε = 2.5 eV-sec. The transverse beam size is given by the normalized 95% emittance of $\varepsilon = 24\pi\times10^{-6}$m rad.

Guided by the previous study*, the transverse stability during the coalescing process seems to be limited by the coupled bunch resistive wall instability, while the single bunch stability is dominated by the slow head-tail instability. Here, we will briefly summarize the upgrade limitations imposed by the above instabilities.

During the coalescing process (at 150 GeV) a train of closely spaced bunches may exhibit an analog of a coupled bunch instability driven by the resistive wall impedance. Further analysis† shows that indeed a rapidly growing envelope of the coherent betatron motion develops along the coalesced train of five bunches. Fig.1 shows the characteristic growth-time of the instability as a function of the betatron tune. It implies that the bunch coalescing process is limited by the growth-time of of 30×10^{-3} sec, which may call for a damping scheme. Here, we will discuss stabilizing effects of the Landau damping induced by both the direct Laslett tune spread and the presence of spool octupoles.

2. Transverse Stability at Low Frequencies

To discuss the dipole mode of the coupled bunch instability we will confine our consideration to the coherent motion characterized by the wavelength, which is much larger than the bunch length or the spacing between bunches. Following classical stability diagram formalism‡ one can rewrite stability condition as a balance of the following forces represented by the corresponding coherent tune shifts:

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* Coherent Instability Limitations of pp and $p\bar{p}$ Upgrade Scenarios, S.A. Bogacz, FN–469, October 1988

† Coherent Instability Limits – Supplement, S.A. Bogacz, FN–507, March 1989

‡ Transverse Stability Consideration for the SPS Beam in Fixed Target Operation, L. Vos, CERN SPS/86-3

♦ resistive wall - driving term

$$\delta\nu_{res} = \frac{MNecZ_\perp}{8\pi^2\nu E} \quad . \tag{1}$$

Here M is the total number of bunches, N is the bunch intensity, ν is the betatron tune, E denotes total energy of a proton and $Z_\perp$ represents the transverse coupling impedance. The relevant contribution is due to the resistive wall and is given by the following expression

$$Z_\perp(\omega) = \frac{Rc}{b^3}\sqrt{\frac{2\rho\mu}{\omega}} \ , \tag{2}$$

where

$$\omega = \omega_o(m - \nu) \ . \tag{3}$$

Here m is the mode number, ρ is the vacuum pipe resistivity and b represents its radius.

♦ direct Laslett tune spread

$$\delta\nu_{sc} = \frac{9r_pRN\varepsilon_{ox}}{4\ell\gamma^2\varepsilon_N F} \quad . \tag{4}$$

where

$$r_p = \frac{e^2}{4\pi\varepsilon_o m_p c^2} \tag{5}$$

is the classical radius of a proton, R is the machine radius, ε_N represents the normalized transverse emittance, F is the dipole mode form-factor and ε_{ox} is the transverse space-charge parameter defined as follows

$$\varepsilon_{ox} = \frac{b^2}{a(a+b)} \ , \tag{6}$$

where b is the vacuum pipe radius and the beam radius a is introduced through the following formula

$$a = \sqrt{\frac{\varepsilon_N}{\pi \langle\beta\rangle \gamma\beta}} \;. \tag{7}$$

Here $\langle\beta\rangle$ denotes the average beta function. Furthermore, the half bunch length, ℓ, is given by the following expression

$$\ell = c\sqrt{\frac{\varepsilon_l}{e\omega_o}}\,\sqrt[4]{\frac{2\eta}{\pi h V E}} \;, \tag{8}$$

where ε_l is the longitudinal emittance, h is the harmonic number and V is the rf voltage amplitude.

♦ pure octupole tune spread

$$\delta\nu_{oct} = \frac{3\langle\beta\rangle^2 \varepsilon_N S_{oct}}{8\pi\gamma F} \;, \tag{9}$$

where S_{oct} is the normalized octupole strength defined as follows

$$S_{oct} = \frac{1}{B\rho} B_o \int b_3 \, ds \;, \tag{10}$$

where for one degree of freedom the following expanssion defines b_3 coefficient

$$B = B_o(b_1 x + b_2 x^2 + b_3 x^3 + ...) \;. \tag{11}$$

Furthermore, for the 0-th harmonics focusing spool octupole (used in the Tevatrons lattice), b_3 as a function of a current passing through the coil, I, is given by the following formula#

$$b_3 = \frac{NI}{6\pi E} \times 10^{11} \quad , \tag{12}$$

where N is the number of spool octupoles and the current, I, is expressed in Ampers.

Finally, the overall stability condition is expressed by the following inequality

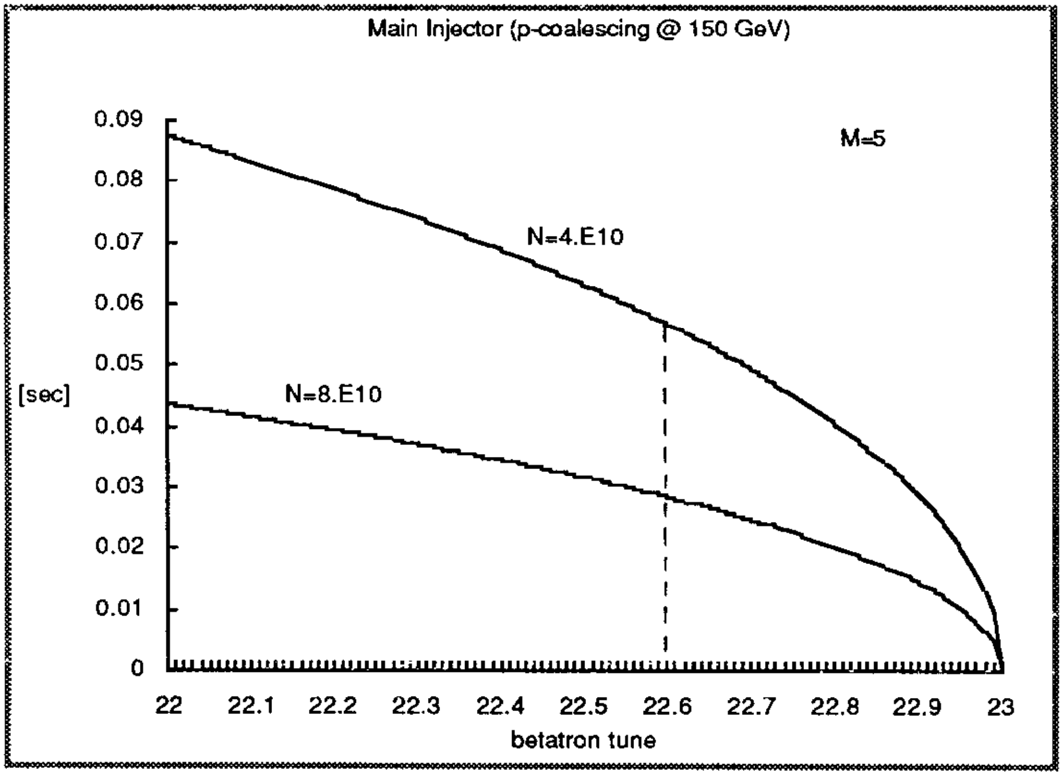
$$\delta\nu_{res} \leq \delta\nu_{sc} + \delta\nu_{oct} \quad , \tag{13}$$

which expresses balance between the driving coherent wake-field and the suppressing Landau damping terms of an ensemble of coupled betatron oscillators. Assuming twelve octupoles, each carrying a current of 1 Amper, one can use Eqs. (1) – (13) to evaluate the overall stability limit of the Main Injector during the bunch coalescing process. Our calculation also assumes bunch intensity $N = 8\times10^{10}$ ppb, betatron tune ν = 22.6 and the average beta function $\langle\beta\rangle$ = 30m. Fig.2 illustrates all three tune shifts given by Eqs. (1), (4) and (9) as a function of energy. Similarly Fig.3 shows these tune shifts as a function of the betatron tune, where the coalescing energy is taken at 150 GeV.

3. Summary

One can see from Fig.2 that in low energy region the space-charge alone provides enough Landau damping to suppress the resistive wall coupled bunch instability. Furthermore, $\delta\nu_{sc}$ drops faster than $\delta\nu_{res}$, with increasing energy, therefore at some energy the lowest frequency mode, $\nu\omega_o$, becomes unstable and one needs some additional tune spread to damp the instability. Figs.2 and 3 show that the necessary tune spread can easily be provided by a set of twelve spool octupoles carrying relatively small currents. They are sufficient enough to provide this extra Landau damping up to the extraction energy of 150 GeV.

Amplitude Dependence of the Tune Shift, N. Gelfand, TM-1393, March 1986



Main Injector (p-coalescing @ 150 GeV)
M=5
N=4.E10
N=8.E10
[sec]
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
22
22.1
22.2
22.3
22.4
22.5
22.6
22.7
22.8
22.9
23
betatron tune

Fig.1

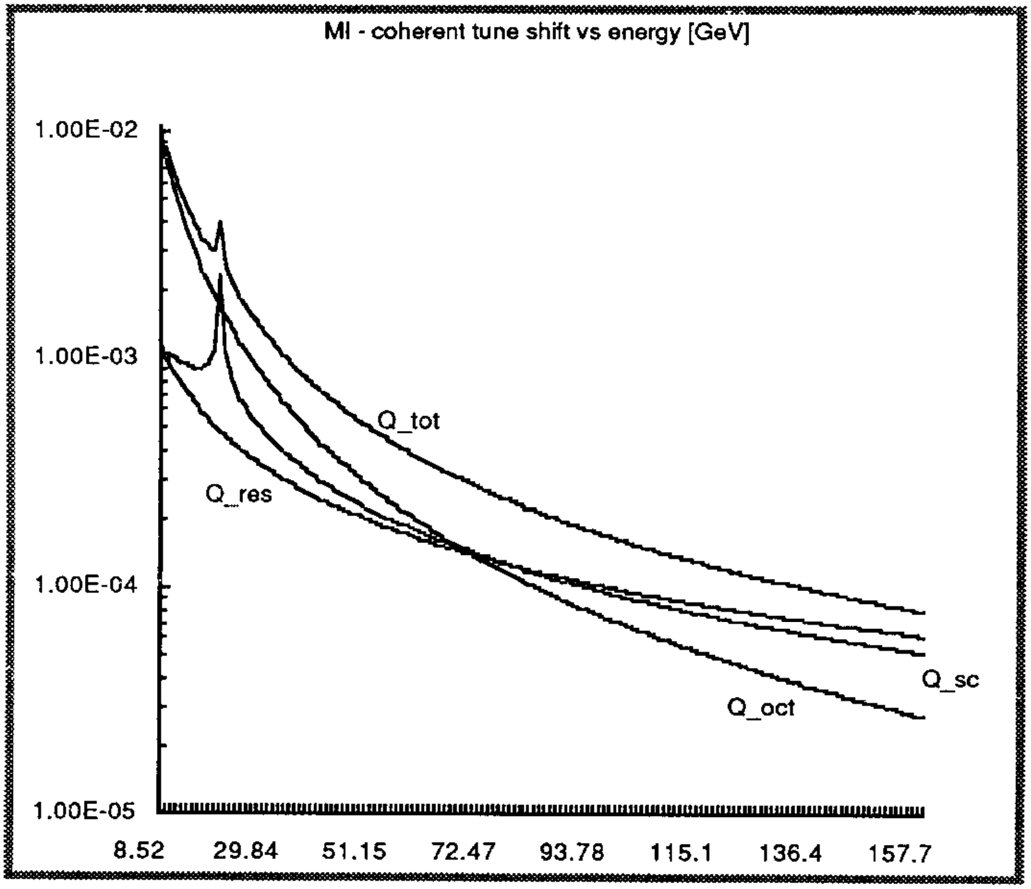

MI - coherent tune shift vs energy [GeV]
1.00E-02
1.00E-03
1.00E-04
1.00E-05
8.52
29.84
51.15
72.47
93.78
115.1
136.4
157.7
Q_tot
Q_res
Q_sc
Q_oct

Fig.2

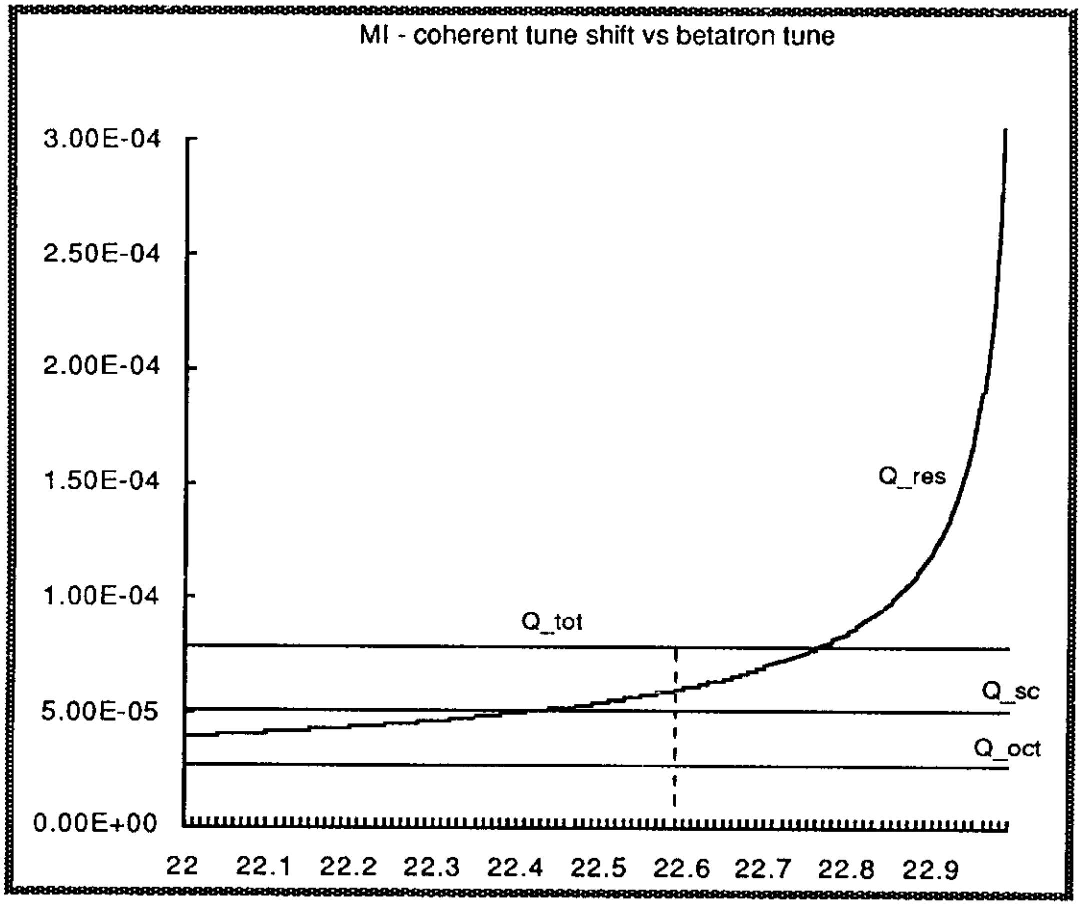

MI - coherent tune shift vs betatron tune
3.00E-04
2.50E-04
2.00E-04
1.50E-04
1.00E-04
5.00E-05
0.00E+00
22
22.1
22.2
22.3
22.4
22.5
22.6
22.7
22.8
22.9
Q_res
Q_tot
Q_sc
Q_oct

Fig.3